IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

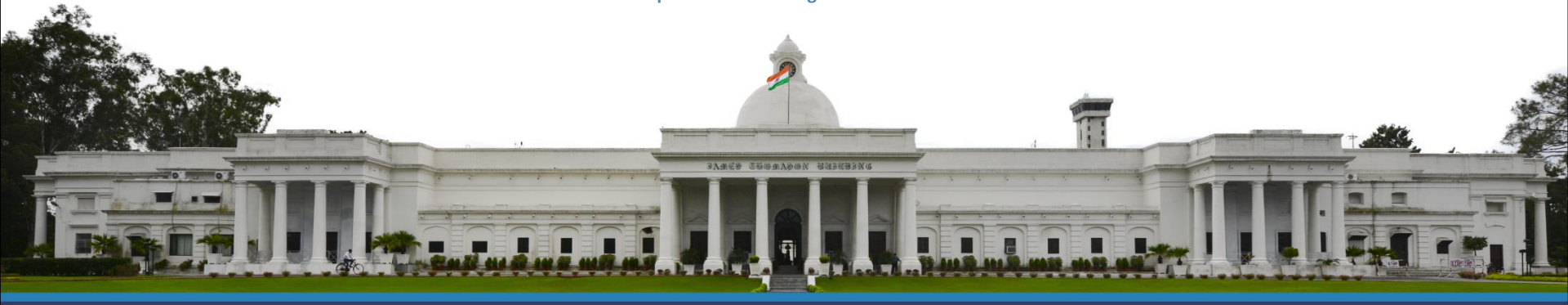
QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 13

Yield to Maturity

J P Singh

Department of Management Studies



COUPON BONDS YIELD CURVE

WHAT IS YTM?

- YTM is the discount rate that equates the present value of future cash flows from the instrument to its current market price (including accrued interest).

$$P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}$$

OBSERVATION

- For a given price, the YTM depends on relative distribution of cash flows.
- In other words, YTM depends on:
- the magnitude of the cash flows and
- the timing of the cash flows.

YTM & RISKINESS

- For a given bond, YTM depends on the riskiness of the cash flows since the price will incorporate the riskiness in the realization of these cash flows.
- YTM is a risk adjusted discount rate that encapsulates the market's perception of the riskiness of the realizability of the cash flows from the instrument.

- Therefore, two bonds with identical cash flows may trade at different YTMs if their risk as perceived by the market is different because the price will adjust accordingly.
- The more risky bond would quote at a lower price and hence a higher YTM.

CONVERSE

- If two bonds carry similar risk perceptions in the market, they would have the same YTM notwithstanding that their cash flow patterns are different.
- The prices would adjust to make the YTMs equal.

YTM: AN INTERPRETATION

- The YTM may be interpreted as some kind of average for the term structure of market interest rates.
- Captures the <u>term structure</u> in one single interest rate.

For annual coupons,

$$P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+S_{0t})^t} = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}$$

SPOT RATES, YTM & PRICE

- There is 1-1 correspondence between price and YTM of a given bond and given spot rates.
- A given spectrum of spot rates uniquely determine the price of a given bond and, therefore, the YTM.

For annual coupons,

$$P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+S_{0t})^t} = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}$$

CAVEAT

- The above correspondence between price and YTM assumes a conventional cash flow pattern wherein a cash outflow (price) is followed by a sequence of cash inflows.
- However, if an instrument involves intermediate mixing of outflows with inflows i.e. cashflows have multiple sign changes, we could have multiple YTMs for a given price.

EXAMPLE

- Consider an investment with the following stream of annual cash flows:
- (0,-1); (1,4); (2,-3)
- The YTM for this stream returns two values:
- y=0% and y=200%

- While there is a unique YTM for a given price for a conventional bond, there is no restriction on
- two bonds with identical cash flow patterns trading at different prices and hence having different YTMs if they have different risk profiles or.
- two bonds with different cash flow patterns trading at the same prices having different YTMs.

YTM & COUPON RATE

- It may be noted that YTMs will not depend on coupon rates since the prices will adjust corresponding to different coupon rates keeping YTM unchanged. However, YTMs depend on the riskiness of the bond.

For annual coupons,

$$P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+S_{0t})^t} = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t}$$

IMPLIED ASSUMPTIONS OF YTM

- THE REINVESTMENT RATE EQUALS THE YTM
- The reinvestment rate is embedded endogenously in the formula itself. No external input.
- THE INVESTOR PLANS TO HOLD THE BOND TO MATURITY
- The maturity value and time are fixed, being elucidated in the contract of issue.

ISSUES WITH YTM: REINVESTMENT RATE

- The YTM measure for long-term maturity coupon bonds tells little about the potential return that an investor may realize if the bond is held to maturity.

ISSUES WITH YTM CONTD…

- Which is better if the reinvestment rate is 6.66% pa?

	Bond A	Bond B
Coupon	10%	3%
Face Value	100	100
Price	138.90	70.22
Maturity	15 years	15 years
Frequency of payment	Annual	Annual
Yield to maturity	6%	6.1%

		A	B
REINVESTMENT RATE		6.66%	6.66%
REINVESTED COUPONS		243.6823	73.10469
REDEMPTION VALUE		100	100
TOTAL		343.6823	173.10469
PRICE		138.9	70.22
EAY		1.0622588	1.0619968

PRICE	A	138.9	B	70.22		
REINV RATE	TOTAL CASH FLOW		TCF/PRICE		RETURN	
	A	B	A	B	A	B
0.06	332.88	170.15	2.39654	2.4231	0.06	0.0608
0.061	334.66	170.68	2.40936	2.43065	0.0604	0.061
0.065	341.94	172.87	2.46177	2.46183	0.0619	0.0619
0.066	343.682	173.1047	2.47431	2.46518	0.0622	0.06212
0.07	351.41	175.71	2.52995	2.50228	0.0638	0.0631

ISSUES WITH YTM CONTD.

- THE YTM OF A PORTFOLIO IS NOT EQUAL TO THE WEIGHTED AVERAGE YTM OF ITS CONSTITUENTS

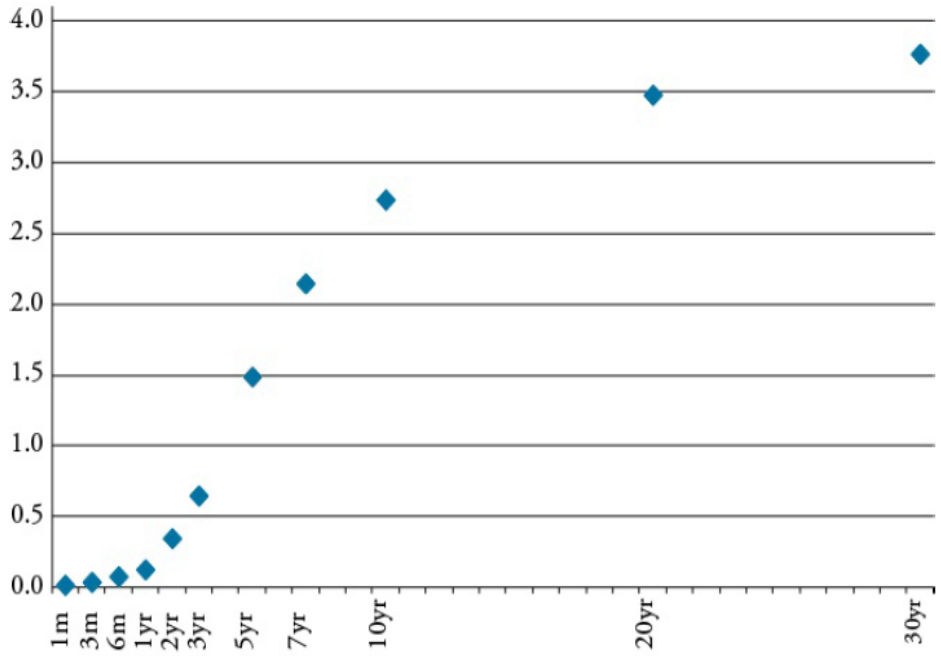
COUPON, YTM & PRICE

- Bond prices and YTM are inversely related.
- The value of a bond would be equal to its face value only when the coupon interest rate and YTM are equal.
- The value of a bond would be higher (lower) than its face value when the coupon interest rate is higher (lower) than YTM.

YIELD CURVE FOR COUPON BONDS

- A yield curve for coupon bonds shows the YTMs for coupon bonds at various terms to maturity e.g. 1-year T-bond, 2-year T-bond etc.
- Please note coupon rate is irrelevant,
- YTMs will not depend on coupon rates since (for a given risk) the prices will adjust corresponding to different coupon rates keeping YTM unchanged.
- However, YTMs depend on the riskiness of the bond.

- **Yields are calculated for several maturities and yields for bonds with maturities between these are estimated by linear interpolation.**



1 month	0.02	5 year	1.49
3 month	0.04	7 year	2.15
6 month	0.08	10 year	2.74
1 year	0.13	20 year	3.48
2 year	0.35	30 year	3.77
3 year	0.65		

The figure shows an upward sloping yield curve for coupon Treasury bonds constructed from yields on 1-month, 3-month, 6- month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 20 year, and 30-year maturities.

PAR BOND YIELD CURVE

- The par yield for a certain bond maturity is the coupon rate that causes the bond price to equal its par value.
- Alternatively, it can be viewed as the YTM of a par bond at each maturity.

- Consider a T-year annual-pay bond and spot rates for one, two, …, T years of S_{01}, S_{02},…,S_{0T}. Then, the coupon rate c necessary for the bond to be trading at par can be worked out from:

$$F = \sum_{t=1}^{T} \frac{cF}{\left(1+S_{0t}\right)^{t}} + \frac{F}{\left(1+S_{0T}\right)^{T}}$$

- c constitutes the par yield on this bond.
- A plot of this par yield with maturity is a par curve.

- A par bond yield curve, or par curve, is not calculated from yields on actual bonds but is constructed from the spot curve.

EXAMPLE

- **Consider the following spectrum of spot rates: $S_{01} = 3\%; S_{02} = 4\%; S_{03} = 5\%$. Calculate the corresponding par rate.**

SOLUTION

- $\frac{c}{1.03} + \frac{c}{1.04^2} + \frac{c+100}{1.05^3} = 100$
- $0.97087c + 0.92456c + 0.86384c = 13.61624$
- $c = 4.9347\%$

- **When the zero curve is upward-sloping, the spot rate for a particular maturity is greater than the par yield for that maturity.**

FORWARD RATES

- Forward rates are yields for future periods.
- A forward rate is a borrowing/lending rate for a loan to be made at some future date.

FORWARD RATE YIELD CURVE

- A forward yield curve shows the future rates for bonds or money market securities for the same maturities for annual periods in the future.
- Typically, the forward curve would show the yields of 1-year securities for each future year, quoted on a semiannual bond basis.

YIELD SPREADS

YIELD SPREADS: DEFINITION

- A yield spread is the difference between the yields of two different bonds, one of which is usually a benchmark bond.
- **Benchmark Spread:** A yield spread relative to a benchmark bond is known as a benchmark spread.
- **G-spread:** A yield spread over a government bond is known as a *G*-spread.
- Yield spreads are typically quoted in basis points.

- For fixed-coupon bonds, on-the-run (most recently issued) government bond yields for the same or nearest maturity are frequently used as benchmarks.

EXAMPLE

- If a 5-year corporate bond has a yield of 6.25% and its benchmark, the 5-year Treasury note, has a yield of 3.50%, the corporate bond has a benchmark spread of 625 – 350 = 275 basis points.

- The benchmark may change during a bond's life.
- For a 5-year corporate bond, when issued, the benchmark spread is stated relative to a 5-year government bond yield, but two years later (when it has three years remaining to maturity) its benchmark spread will be stated relative to a 3-year government bond yield.

YIELD SPREADS: FLOATING RATE BONDS

- **Floating-rate securities typically use LIBOR as a benchmark rate.**

UTILITY OF YIELD SPREADS

- Yield spreads are useful for analyzing the factors that affect a bond's yield.
- If a corporate bond's yield increases from 6.25% to 6.50%, this may have been caused by
- (i) factors that affect all bond yields (macroeconomic factors) or
- (ii) firm-specific or industry - specific (microeconomic) factors.

- If a bond's yield increases but its yield spread remains the same, the yield on its benchmark must have also increased, which suggests macroeconomic factors caused bond yields in general to increase.
- However, if the yield spread increases, it means the benchmark yield has not changed. This suggests the increase in the bond's yield was caused by microeconomic factors such as credit risk or the issue's liquidity.

Z-SPREAD & OAS

Z-SPREAD & OAS

- There are three sources that create uncertainty in the cash flows from investment in a bond.
- Possibility of default by the issuer in payment of interest & principal for straight risky bonds (credit risk).
- In the case of option embedded bonds, there is the additional possibility of exercise of option (option risk).
- Changes in interest rates and hence bond prices (interest rate risk).
- The first source of risk is captured by the Z-spread and the second source by the OAS.

ZERO-VOLATILITY SPREADS (Z-SPREAD) FOR STRAIGHT RISKY BONDS

- G- spread essentially involves comparing the YTM of the given bond with the YTM of a G-bond with corresponding maturity.
- Thus, G-spread assumes that the spot yield curve is flat so that yields are approximately the same across maturities.
- Normally, however, the spot yield curve is upward-sloping (i.e., longer-term yields are higher than shorter-term yields).

- In the Z spread or zero volatility spread, we add an equal amount to each benchmark spot rate and value the bond with those rates.
- When we find an amount which, when added to the benchmark spot rates, produces a value equal to the market price of the bond, we have the appropriate yield curve spread.
- $P_0 = \sum_{t=1}^{T} \frac{C_t}{\left(1+S_{B,0t}+\delta\right)^t}$
- Thus, Z-spread accounts for the shape of the yield curve.